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LETTER TO THE EDITOR

# Nonlinear deformations of su(2) and su(1,1) generalizing Witten's algebra

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**Abstract.** Nonlinear deformations of su(2) and su(1,1) involving two deforming functions $f(J_0)$ and $g(J_0)$ are considered. For $g(J_0) = 1$, they reduce to some algebras first studied by Polychronakos and Roček. Special emphasis is laid on the case where $g(J_0)$ is a linear function of $J_0$. It is shown that for any $\lambda = 2, 3, \ldots$, there exist $(\lambda - 1)$-parameter algebras that are deformations of su(2) or su(1,1) respectively, and for which $f(J_0)$ is a polynomial of degree $\lambda$. For $\lambda = 2$, such algebras are equivalent to Witten's first deformation of su(2) or su(1,1). For any $\lambda$, the spectrum of $J_0$ is exponential instead of linear as in the case where $g(J_0) = 1$.

Quantized universal enveloping algebras, also called $q$-algebras or quantum groups, refer to some specific deformations of Lie algebras (Jimbo 1985, Drinfeld 1986). In recent years, there has been some interest in more general deformations involving an arbitrary real function of the weight generators and including $q$-algebras as a special case (Polychronakos 1990, Roček 1991, Daskaloyannis 1991, Granovskii *et al* 1992). As shown by Roček for the deformed su(2) algebra, the presence of an arbitrary function gives rise in the representation theory to a wealth of interesting phenomena that are absent in the $q$-algebra case and might prove useful in some applications to physical models.

The purpose of the present letter is to further extend the class of nonlinear algebras considered so far by allowing for two deforming functions instead of one. In the case of the deformed su(2) and su(1,1) algebras to which we shall restrict ourselves here, the two deforming functions $f(J_0)$ and $g(J_0)$ appear in the commutator of $J_+$ with $J_-$ and in that of $J_0$ with $J_+$ or $J_-$, respectively. The definition of the algebras is completed by the condition that there does exist a Casimir operator of the type considered by Polychronakos (1990) and Roček (1991). For $g(J_0) = 1$, one finds the algebras previously studied, for which $f(J_0)$ may be arbitrary. On the contrary, whenever $g(J_0)$ explicitly depends upon $J_0$, the existence of a Casimir operator imposes some restrictions upon $f(J_0)$.

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Special emphasis will be laid on the simplest non-trivial case, namely that corresponding to linear $g(J_0)$. Such a case looks interesting because the spectrum of $J_0$ is exponential instead of linear as for the algebras studied so far. The algebras herein considered might therefore be applicable to problems wherein such a type of spectrum makes its appearance (Fairlie and Nuyts 1991, Spiridonov 1992a, b). An example is provided by the so-called logarithmic trajectories in quantum field theory (Nuyts and Cremmer 1971).

For linear $g(J_0)$, the allowed functions $f(J_0)$ are polynomials of degree higher than 1. When $f(J_0)$ is quadratic, one gets in the su(2) case a quadratic algebra known as Witten's first deformation of su(2) (Witten 1990, Curtright and Zachos 1990). The families of algebras introduced in the present letter may therefore be considered as higher-degree generalizations of Witten's algebra. Their representation theory may be investigated by simple methods extending those used in standard angular momentum theory, as was done by Roček (1991) for $g(J_0) = 1$.

Let us introduce two sets of algebras that are nonlinear deformations of su(2) and su(1,1) and denote them by $\mathcal{A}^+$ and $\mathcal{A}^-$ respectively (in the following, except otherwise stated, the upper sign corresponds to $\mathcal{A}^+$ and the lower one to $\mathcal{A}^-$).

*Definition.* Let $\mathcal{A}^\pm$ denote the associative algebras over $\mathbb{C}$ generated by three operators $J_0 = (J_0)^\dagger$, $J_+$, and $J_- = (J_+)^\dagger$, satisfying the commutation relations

$$[J_0, J_+] = g(J_0)J_+ \qquad [J_0, J_-] = -J_-g(J_0) \qquad [J_+, J_-] = \pm f(J_0) \qquad (1a,b,c)$$

where $f(J_0)$ and $g(J_0)$ are two real functions of $J_0$, holomorphic in the neighbourhood of zero,

$$f(J_0) = \sum_{i=1}^{\infty} \alpha_i J_0^i \qquad g(J_0) = \sum_{i=0}^{\infty} \beta_i J_0^i \qquad (2)$$

and satisfying the following two conditions:

(i) the algebras have a Casimir operator of the type

$$C = J_-J_+ + h(J_0) \qquad (3)$$

where $h(J_0)$ is some real function of $J_0$, holomorphic in the neighbourhood of zero,

$$h(J_0) = \sum_{i=1}^{\infty} \gamma_i J_0^i \qquad (4)$$

(ii) for some values of the parameters, $f(J_0)$ and $g(J_0)$ go to $\pm 2J_0$ and 1 respectively.

*Remarks.* (i) It is easy to check that the algebras are well defined, i.e. that the Jacobi identity is satisfied by $J_0$, $J_+$ and $J_-$.

(ii) Equations (1*a,b*) and (3) can be rewritten in the equivalent forms

$$(J_0 - g(J_0))J_+ = J_+J_0 \qquad J_-(J_0 - g(J_0)) = J_0J_- \qquad (5a,b)$$

and

$$C = J_+J_- + h(J_0) - f(J_0) = \tfrac{1}{2}((2 \mp \gamma_1)J_-J_+ \pm \gamma_1 J_+J_- + 2h(J_0) \mp \gamma_1 f(J_0)) \tag{6}$$

where the symmetrized form of $C$ does not contain any linear term in $J_0$ as in the corresponding expression $C = \frac{1}{2}(J_-J_+ + J_+J_-) \pm J_0^2$ for su(2) and su(1,1).

(iii) It is always possible to renormalize $J_0$, $J_+$ and $J_-$ so as to obtain $\alpha_1 = \pm 2$ and $\beta_0 = 1$. In the following, we shall assume that such a renormalization has been carried out.

(iv) For $g(J_0) = 1$, we obtain the algebras introduced by Polychronakos (1990) and Roček (1991), while for $g(J_0) = 1$ and $f(J_0) = \pm\sinh(\eta J_0)/\sinh(\eta/2)$ or $\pm\sin(\eta J_0)/\sin(\eta/2)$, we get the standard $su_q(2)$ and $su_q(1,1)$ algebras, where $q = \exp\eta$ or $\exp(i\eta)$ respectively.

It is now straightforward to prove the following.

*Proposition 1.* The operator $C$, defined in (3), commutes with $J_0$, $J_+$, and $J_-$, if and only if the functions $f(J_0)$, $g(J_0)$ and $h(J_0)$ satisfy the relation

$$h(J_0) - h(J_0 - g(J_0)) = f(J_0). \tag{7}$$

In the remainder of this letter, we shall restrict ourselves to those algebras for which $f(J_0)$, $g(J_0)$, and $h(J_0)$ are polynomials of degree $\lambda$, $\mu$, and $\nu$, respectively. We shall now proceed to show that for some definite values of $\lambda$, $\mu$, $\nu$, such algebras do exist, which amounts to proving that equation (7) admits a solution.

Let us first consider the case where $\mu = 0$, corresponding to the Polychronakos–Roček algebras.

*Proposition 2.* For $g(J_0) = 1$ and $f(J_0)$ a generic $\lambda$-degree polynomial, a solution of (7) is given by

$$h(J_0) = \sum_{i=1}^{\lambda} \frac{(-1)^{i+1}}{i+1} \alpha_i (B_{i+1}(-J_0) - B_{i+1}) \tag{8}$$

where $B_n(x)$ and $B_n$ denote Bernoulli polynomials and Bernoulli numbers respectively.

*Proof.* Consider (7) for $g(J_0) = 1$ and write $h(J_0)$ as $h(J_0) = \sum_{i=1}^{\lambda} \alpha_i h_i(J_0)$. Since for generic $f(J_0)$, the parameters $\alpha_i$ are independent, the functions $h_i(J_0)$ have to satisfy the recursion relations

$$h_i(J_0) - h_i(J_0 - 1) = J_0^i \qquad i = 1, \ldots, \lambda. \tag{9}$$

Comparison of (9) with the recursion relation of Bernoulli polynomials (Erdélyi *et al* 1953) determines $h_i(J_0)$ up to some additive constant, which can be found by using the condition $h_i(0) = 0$. □

Consider next the cases where $\mu \geqslant 1$.

*Proposition 3.* If $f(J_0)$ and $g(J_0)$ are polynomials of degree $\lambda$ and $\mu$ respectively, where $\lambda$, $\mu \geqslant 1$ and $\beta_1 \neq 2$, then a polynomial solution $h(J_0)$ of (7) cannot exist if $\lambda \neq \mu\nu$ or if $f(J_0)/g(J_0)$ is not a polynomial.

*Proof.* For $h(J_0) = \sum_{i=1}^{\nu} \gamma_i J_0^i$, equation (7) becomes

$$g(J_0)\sum_{i=1}^{\nu} \gamma_i (J_0^{i-1} + J_0^{i-2}(J_0 - g(J_0)) + \cdots + (J_0 - g(J_0))^{i-1}) = f(J_0) \tag{10}$$

where the sum over $i$ is a polynomial of degree $\mu(\nu-1)$ whenever the conditions on $f(J_0)$ and $g(J_0)$ stated in the proposition are fulfilled. □

Whenever $\lambda = \mu\nu$ and $f(J_0)$ can be written as

$$f(J_0) = g(J_0) \sum_{i=1}^{\mu(\nu-1)} \xi_i J_0^i \tag{11}$$

where $\xi_i$ are some real parameters and $\xi_1 = \pm 2$ (because $\alpha_1 = \pm 2$ and $\beta_0 = 1$), equation (10) can be easily solved for small values of $\nu$ by equating the coefficients of equal powers of $J_0$ on both sides. Note that the terms independent of $J_0$ lead to the condition

$$\sum_{i=1}^{\nu} (-1)^{i-1} \gamma_i = 0 \tag{12}$$

showing that $h_\nu(-1) = 0$; hence $h(J_0)$ always contains a factor $J_0(J_0 + 1)$.

For $\nu = 2$, for instance, a solution of (10) does exist whenever $\beta_1 \neq 2$ and $\xi_2$, $\ldots$, $\xi_\mu$ are given in terms of $\beta_1, \ldots, \beta_\mu$ by the relations $\xi_i = \mp 2\beta_i/(2 - \beta_1)$, $i = 2$, $\ldots$, $\mu$. Hence, we get the following result.

*Proposition 4.* There exist $\mu$-parameter algebras $A^{\pm}_{\beta_1\beta_2\ldots\beta_\mu}(2\mu, \mu)$, corresponding to $\mu = 1, 2, \ldots$, $\lambda = 2\mu$, $\nu = 2$, $\beta_1 \neq 2$, for which

$$f(J_0) = \pm 2 J_0 g(J_0)\left(1 - \frac{1}{2 - \beta_1} \sum_{i=2}^{\mu} \beta_i J_0^{i-1}\right) \tag{13a}$$

$$h(J_0) = \pm \frac{2}{2 - \beta_1} J_0(J_0 + 1). \tag{13b}$$

Let us now consider the case where $\mu = 1$ and $\lambda = \nu$ in more detail. Then

$$g(J_0) = 1 + (1 - q)J_0 \tag{14}$$

depends upon a single parameter $q$ (or equivalently $\beta_1 = 1 - q$), which will be assumed real and positive. The value $q = 1$ corresponds to the undeformed algebras.

*Proposition 5.* There exist $(\lambda-1)$-parameter algebras $\mathcal{A}^{\pm}_{\alpha_2\alpha_3\ldots\alpha_{\lambda-1}q}(\lambda,1)$, corresponding to $\lambda=\nu=2,3,\ldots,\mu=1$, $q\in\mathbb{R}^+$, for which $g(J_0)$ is given by (14) and $f(J_0)$, $h(J_0)$ by

$$f(J_0)=J_0 g(J_0)\sum_{i=1}^{\lambda-1}\alpha_i J_0^{i-1}\sum_{j=0}^{\lambda-i-1}(-1)^j(1-q)^j J_0^j \tag{15a}$$

$$h(J_0)=J_0(J_0+1)\sum_{i=1}^{\lambda-1}\alpha_i\sum_{j=0}^{\lambda-2}\left(\sum_{k=1}^{j+1}(-1)^{j+1-k}A^{-1}_{k,i+1}\right)J_0^j \tag{15b}$$

respectively. Here $\mathbf{A}^{-1}$ denotes the inverse of the $\lambda\times\lambda$ matrix $\mathbf{A}$ whose elements are

$$A_{ij}=\begin{cases}(-1)^j & \text{if } i=1\\ \delta_{i,j+1}-(-1)^{j-i+1}\binom{j}{i-1}q^{i-1} & \text{if } i=2,\ldots,\lambda\end{cases} \tag{16}$$

where $\binom{j}{i-1}$ is a binomial coefficient and $\binom{j}{i-1}\equiv 0$ if $j<i-1$.

*Proof.* When equation (14) is introduced into (7), $f(J_0)$ and $h(J_0)$ are expanded into powers of $J_0$ and equal powers are equated on both sides of the transformed equation, one obtains a system of $\lambda$ equations in $\lambda$ unknowns $\gamma_i$, $i=1,\ldots,\lambda$,

$$\sum_{j=1}^{\lambda}A_{ij}\gamma_j=\alpha_{i-1}\qquad i=1,\ldots,\lambda \tag{17}$$

plus an extra condition

$$(1-q^\lambda)\gamma_\lambda=\alpha_\lambda. \tag{18}$$

In (17), $A_{ij}$ is defined by (16) and $\alpha_0\equiv 0$.

The determinant of $\mathbf{A}$ is the same as that of $\mathbf{A}'$, whose elements are defined by

$$A'_{ij}=\begin{cases}A_{1j} & \text{if } i=1\\ \sum_{k=0}^{i-2}(-1)^k(1-q)^k A_{i-k,j}+(-1)^{i-2}(1-q)^{i-1}A_{1j} & \text{if } i=2,\ldots,\lambda.\end{cases} \tag{19}$$

Since $\mathbf{A}'$ is an upper triangular matrix with diagonal elements $A'_{ii}$ equal to $-1$ if $i=1$ and to $1+q+\cdots+q^{i-1}$ if $i=2,\ldots,\lambda$, one obtains

$$\det\mathbf{A}=-\prod_{i=1}^{\lambda}(1+q+\cdots+q^i)\neq 0 \tag{20}$$

showing that system (17) has a unique solution

$$\gamma_i=\sum_{j=1}^{\lambda-1}A^{-1}_{i,j+1}\alpha_j\qquad i=1,\ldots,\lambda. \tag{21}$$

By taking the remark following (12) into account, this directly leads to equation (15*b*).

By combining (18) with (21), one finds that $\alpha_\lambda$ is related to $\alpha_1 = \pm 2$, $\alpha_2, \ldots, \alpha_{\lambda-1}$ through the condition

$$\alpha_\lambda = (1-q^\lambda) \sum_{j=1}^{\lambda-1} A^{-1}_{\lambda,j+1} \alpha_j \tag{22}$$

where

$$A^{-1}_{\lambda k} = (-1)^{\lambda-k} \frac{(1-q)^{\lambda-k}}{1+q+\cdots+q^{\lambda-1}} \qquad \text{if } k = 2, \ldots, \lambda. \tag{23}$$

Equation (23) can be proved by a generating function technique. By introducing the function

$$\phi_j(t) = \sum_{i=1}^{\lambda} A_{ij} t^i = \begin{cases} t(t^j - (qt-1)^j + 2(-1)^j) & \text{if } j = 1, \ldots, \lambda - 1 \\ t((qt)^\lambda - (qt-1)^\lambda + 2(-1)^\lambda) & \text{if } j = \lambda \end{cases} \tag{24}$$

into the left-hand side of the equation

$$\sum_{j=1}^{\lambda} A^{-1}_{jk} \phi_j(t) = t^k \tag{25}$$

one indeed obtains that

$$\psi_k(t) = \sum_{j=1}^{\lambda} A^{-1}_{jk} t^j \tag{26}$$

satisfies the relation

$$\psi_k(t) - \psi_k(qt-1) + 2\psi_k(-1) - A^{-1}_{\lambda k}(1-q^\lambda)t^\lambda = t^{k-1} \tag{27}$$

where, from $\psi_k(0) = 0$, it results that $\psi_k(-1) = \delta_{k,1}$. For $t = (q-1)^{-1}$ and $k > 1$, equation (27) directly leads to (23). After a straightforward calculation, equation (15*a*) is finally derived. □

The algebras $\mathcal{A}^{\pm}_{\alpha_2 \ldots \alpha_{\lambda-1} q}(\lambda, 1)$ have an interesting symmetry property, which allows us to restrict the $q$ values to the interval $0 < q < 1$.

*Proposition 6.* The transformation $q = q'^{-1}$, $\alpha_i = (-q')^{1-i}\alpha'_i$, $J_0 = -q' J'_0$, $J_\pm = \sqrt{q'} J'_\mp$ maps the algebras $\mathcal{A}^{\pm}_{\alpha_2 \ldots \alpha_{\lambda-1} q}(\lambda, 1)$ onto $\mathcal{A}^{\pm}_{\alpha'_2 \ldots \alpha'_{\lambda-1} q'}(\lambda, 1)$.

For the first few $\lambda$ values, the explicit expressions of $f(J_0)$ and $h(J_0)$, obtained by inverting the matrix **A**, are given by

$\mathcal{A}^{\pm}_q(2, 1)$

$$f(J_0) = \pm 2 J_0 (1 + (1-q) J_0) \tag{28a}$$

$$h(J_0) = \pm \frac{2}{1+q} J_0 (J_0 + 1) \tag{28b}$$

$\mathcal{A}^{\pm}_{pq}(3,1)$

$$f(J_0) = \pm 2J_0(1+(1-q)J_0)(1-(1-p)J_0) \tag{29a}$$

$$h(J_0) = \pm\frac{2}{(1+q)(1+q+q^2)}J_0(J_0+1)(1+(p+q)q-(1-p)(1+q)J_0) \tag{29b}$$

$\mathcal{A}^{\pm}_{p_1p_2q}(4,1)$

$$f(J_0) = \pm 2J_0(1+(1-q)J_0)(1-(1-p_1)J_0+(1-p_2)J_0^2) \tag{30a}$$

$$\begin{aligned} h(J_0) = \pm&\frac{2}{(1+q+q^2)(1+q+q^2+q^3)}J_0(J_0+1)(1-q+3q^2+q^4 \\ &+p_1q(1+q^2)+p_2q(1-q)+(-1+q^2-q^3+p_1(1+q+q^2+q^3) \\ &-p_2q(1+2q))J_0+(1-p_2)(1+q+q^2)J_0^2). \end{aligned} \tag{30b}$$

Here we have set $\alpha_2 = \pm 2(p-q)$ if $\lambda = 3$, and $\alpha_2 = \pm 2(p_1 - q)$, $\alpha_3 = \pm 2(q + p_1(1-q) - p_2)$ if $\lambda = 4$. Note that $\mathcal{A}^{\pm}_q(2,1)$ also appear as special cases of the algebras considered in proposition 4, and that $\mathcal{A}^{+}_q(2,1)$ is equivalent to Witten's first deformation of su(2) (Witten 1990, Curtright and Zachos 1990).

As a final point, let us briefly outline the representation theory of the sets of algebras $\mathcal{A}^{\pm}_{\alpha_2\ldots\alpha_{\lambda-1}q}(\lambda,1)$. As for the undeformed algebras su(2) and su(1,1), the commuting Hermitian operators $C$ and $J_0$ may be simultaneously diagonalized.

*Proposition 7.* If $|cm\rangle \neq 0$ is a simultaneous eigenstate of $C$ and $J_0$, corresponding to the eigenvalues $c$ and $m$ respectively, then $J_+^n|cm\rangle$ (respectively $J_-^n|cm\rangle$), $n \in \mathbb{N}^+$, is either the null vector or a simultaneous eigenstate of $C$ and $J_0$, corresponding to the eigenvalues $c$ and $mq^{-n} - (1-q^{-n})/(1-q)$ (respectively $mq^n - (1-q^n)/(1-q)$).

*Proof.* From (5*a*,*b*), it results that the statement is true for $n = 1$. The general result is proved by induction over $n$. □

*Remark.* Contrary to what happens for su(2), su(1,1) and their deformations corresponding to $g(J_0) = 1$, the spectrum of $J_0$ is not linear, but exponential. Exponential spectra had already been found by Fairlie and Nuyts (1991) in connection with alternative quantization schemes, and by Spiridonov (1992a, b) in the framework of $q$-deformed supersymmetric quantum mechanics.

Another difference with the undeformed algebras lies in the domain of variation of $m$.

*Proposition 8.* If $m$ belongs to the interval $((q-1)^{-1}, +\infty)$ (respectively $(-\infty, (q-1)^{-1})$), then all the eigenvalues of $J_0$ obtained by successive applications of $J_+$ or $J_-$ upon $|cm\rangle \neq 0$ belong to the same interval and $J_+$ (respectively $J_-$) is a raising generator. If $m = (q-1)^{-1}$, then neither $J_+$ nor $J_-$ change the $J_0$ eigenvalue.

*Proof.* According to proposition 7, it is enough to show that if $m < (q-1)^{-1}$, $m = (q-1)^{-1}$, or $m > (q-1)^{-1}$, then such is the case for $q^{-1}(m+1)$ and $qm - 1$. □

The unitary irreducible representations (unirreps) of $\mathcal{A}^{\pm}_{\alpha_2\ldots\alpha_{\lambda-1}q}(\lambda, 1)$ may fall into one of four classes: (i) infinite-dimensional unirreps with a lower bound $-j$; (ii) infinite-dimensional unirreps with an upper bound $J$; (iii) infinite-dimensional unirreps with neither lower nor upper bounds; and (iv) finite-dimensional unirreps with both lower and upper bounds, $-j$ and $J$ (where in general $j \neq J$). In addition, there is a trivial one-dimensional unirrep corresponding to $m = (q-1)^{-1}$. In forthcoming publications (Delbecq and Quesne 1993a, b), the conditions for the existence of the various types of unirreps will be discussed for some algebras of the family.

An open question concerns the existence of a comultiplication rule that would enable us to endow the algebras with a Hopf algebraic structure. It is important to solve this problem if we want to define a product of two independent representations of a given algebra that is still a representation of this algebra. In principle, we could use a deforming functional mapping su(2) or su(1,1) onto the algebra (Curtright and Zachos 1990) to induce the coproduct from the ordinary addition rule for su(2) or su(1,1) (Polychronakos 1990, Curtright *et al* 1991). However, as such a procedure leads to complicated and untractable coproducts, a direct explicit determination would be preferable (Granovskii *et al* 1992).

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